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Technical Note

Limiting cross-flow velocity below which heat flux is determined by natural convection laws

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Abstract

In estimating heat flux or heat transfer rates in technological or meteorological applications, we propose here that a parameter of considerable interest is the limiting value of the cross-flow velocity upto which the heat flux is given by the free convection laws to a sufficiently good approximation. The question of determining this limiting value is addressed here by analysing available data in three geometries: sphere, flat plate and cylinder. It is found that, in each case, the limiting velocity can be concisely and elegantly expressed in terms of the 'internal' Froude number, which is related to a parameter introduced by Klyachko [Trans. ASME J. Heat Transfer 85 (1963) 355] in his work on heat transfer from spheres. We find that departure of the heat flux from the value for natural convection (to within 5%) occurs when the internal Froude number exceeds a limiting value, which is found to be about 0.063 for sphere, about 1.93 for flat plate and about 1.65 for cylinder. The critical cross-flow velocity upto which natural convection provides a good approximation thus appears to be very much larger in 2D than in 3D flows for comparable characteristic length scales.

1. Introduction

A heated body in still fluid loses heat by natural or free convection—a phenomenon which has been studied extensively. The results of detailed heat transfer measurements in natural convection are usually presented in terms of the Nusselt number as a function of the Rayleigh number (e.g. [2–4]). If the heated body is in a cross-flow, then the regime changes first to mixed convection and then to forced convection as cross-flow velocities increase. This note concerns the cross-flow velocity at which the regime departs from natural to mixed convection—a subject on which there are few investigations, especially when the flow is turbulent.

Knowledge of the cross-flow velocity upto which heat transfer coefficients may be given by theories of natural convection has obvious importance in engineering applications. However, the issue has also acquired considerable importance recently in atmospheric problems, especially in tropical latitudes, following the finding of Miller et al. [5] that atmospheric general circulation models perform much better in the tropics, in particular for simulations of the Indian monsoons, with an enhancement of low wind fluxes over the values given by the well known Monin–Obukhov theory [6,7]. The present work analyses the available engineering data in the light of the work of Narasimha and Rao [8] in what they call the weakly forced convection regime. In this regime they find that the observed heat flux in the atmosphere is independent of wind speed and the drag increases linearly with wind speed. There is some support for these ideas from the earlier work of Ingersoll [9] and Grachev [10] in laboratory flows. It is in this context that we introduce the concept of a limiting cross-flow velocity upto which the laws of natural convection are applicable.

The specific question we ask is the following. Upto what values of cross-flow velocity is the heat flux in (weakly) forced convection characterized (approximately) by the law governing pure natural convection? Although there is data in the engineering literature that

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can be utilized to answer this question, it has not been explicitly considered in the well-known treatments of the subject [2–4].

2. The available data

Extensive data are available for only three geometries: sphere, flat plate and cylinder. Fortunately these data do provide a consistent answer to the question posed in this note, and further highlight an important difference between 2D and 3D flows.

2.1. Sphere

Yuge [11] carried out experiments on heat transfer between a heated sphere and an air stream in two wind tunnel configurations (of jet diameter 6.09 and 25.4 cm respectively). The experiments covered the Reynolds number range $Re = 3.5$ to 1.44×10^5, and Grashof number range $Gr = 1$ to 10^5, where $Re = UD/\nu_m$, $Gr = (gD^3\Delta T)/T_f\nu_m^2$, U is the free stream cross-flow velocity, D is the diameter of the sphere, ν_m is kinematic viscosity evaluated at the film temperaure $T_f = (T_0 + T_\infty)/2$, and ΔT is the temperature difference $T_0 - T_\infty$ between the temperature T_0 at the surface of the body and T_∞ in the free stream. Yuge presented his data in the form Nusselt number Nu versus the square root of the Reynolds number for fixed Grashof numbers; here $Nu = QD/k\Delta T$, where Q is the heat flux and k the thermal conductivity.

Fig. 1a is a replot of Yuge's experimental data in the form of Nu/Nu_n versus the Reynolds number Re, where Nu_n is the value that the Nusselt number would have in natural convection for the given temperature differential. It is seen that at $Gr = 397$, Nu departs from Nu_n only beyond $Re = 4$, but at $Gr = 1819$ the departure occurs at $Re = 7$, so higher the Grashof number larger the Reynolds number range over which the free convection heat transfer law is valid.

Klyachko [1], analyzing Yuge's data, proposed that

$$Nu/Nu_n = [1 + (Re^2/Gr)]^{1/5}, \qquad (1)$$

where

$$Nu_n = 1.18(Gr\,Pr)^{1/8} \quad \text{for } 10^{-3} < Gr\,Pr < 500,$$
$$= 0.54(Gr\,Pr)^{1/4} \quad \text{for } 500 < Gr\,Pr < 2\times10^7,$$

and $Pr(=\nu/\alpha)$ is the Prandtl number (0.7 for air). An alternative correlation for Yuge's experimental results has been provided by Armaly et al. [12]:

$$(Nu - 2)/(Nu_f - 2) = [1 + (0.795\Omega^{1/4})^{3.5}]^{1/3.5}, \qquad (2)$$

where

$$Nu_f - 2 = 0.493Re^{1/2} \quad \text{(for forced convection)} \qquad (3)$$

and $\Omega = Gr/Re^2$. Using the relation

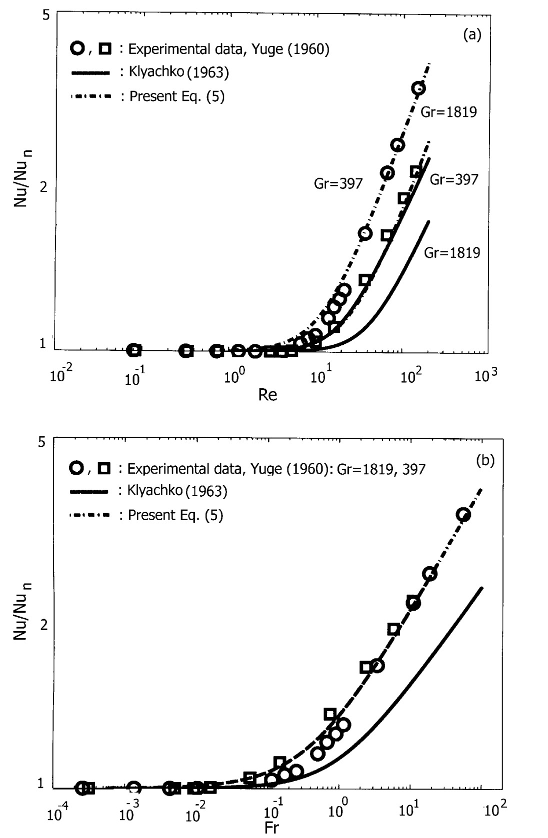

Fig. 1. Nusselt number ratio as a function of Reynolds number (a) and Froude number (b), for sphere in cross-flow.

$$Nu_n - 2 = 0.392Gr^{0.25} \quad \text{(for natural convection)}, \qquad (4)$$

in Eqs. (2) and (3), we derive

$$(Nu - 2)/(Nu_n - 2) = 1.258[1 + (0.795\Omega^{1/4})^{3.5}]^{1/3.5}\ \Omega^{1/4}. \qquad (5)$$

We note that the parameter Ω introduced by Klyachko and Armaly does not contain the viscosity, and is best seen as a version of the well-known Froude number; more precisely it is what Turner [13] defines as 'internal' or 'densimetric' Froude number,

$$Fr = U/(g'D)^{1/2}, \quad g' \equiv g\Delta\rho/\rho_r, \qquad (6)$$

where g is acceleration due to gravity, $\Delta\rho$ a characteristic density differential in the flow and ρ_r a reference density. (In the following Fr will often be just called the Froude number.) Apart from its conciseness and elegance, the chief advantage of using the 'internal' Froude number is that it also covers the case where non-thermal density differences (e.g. due to salinity gradients in water) are responsible for buoyancy. Where the buoyancy forces are purely thermal in origin (as in the present case) we can write

$$Fr \equiv U/(g'D)^{1/2} = U(T_r/|\Delta T|gD)^{1/2},$$

noting that $\Delta\rho/\rho_r = -\Delta T/T_r$ where ρ_r and T_r are reference density and temperature in the flow, and ΔT is a characteristic temperature differential (e.g. between freestream and body surface).

The data of Fig. 1a are now replotted in Fig. 1b, along with the Eqs. (1) and (5), with Fr as the abscissa. It is first of all seen that the two sets of data plot on virtually the same curve, demonstrating that Fr is indeed the appropriate parameter. Somewhat unexpectedly, Klyachko's formula underpredicts the Nusselt number ratio; the reason may be that it was devised to fit the data over a wide Reynolds number range, whereas we are interested only in the beginning of departure from natural convection; this highlights the importance of reexamining the data for the specific purpose of determining the limiting cross-flow velocity of interest here. From Fig. 1b we can define a limiting Froude number, say Fr_0, below which the heat transfer is given by the formula for natural convection. The value of Fr_0 will naturally depend on the criterion adopted for identifying departure of Nu/Nu_n from unity, and this in turn depends on experimental uncertainties. No precise figures for these are given by the authors, especially near $Nu \simeq Nu_n$. If we adopt too low a value for the criterion $(Nu/Nu_n - 1)$ it will be difficult to identify Fr_0 unambiguously from the experimental data, and if we adopt too high a value the heat transfer regime will have changed. A practical optimum appears to be a departure by 5%. Accepting this figure, the criterion is

$$Nu < 1.05 Nu_n \quad \text{if } Fr < (Fr_0 = 0.063).$$

2.2. Flat plate

Wang [14] carried out experiments on heat transfer between a horizontal flat plate (of size 1.2 m (length) × 0.25 m (width) × 0.02 m (thickness)) and an air stream inside an open loop, low speed wind tunnel whose dimensions are 0.25 m × 0.3 m × 1.5 m (width by height by length). The experiments covered the Reynolds number range $Re = 10^3$ to 10^6 and Grashof number range $Gr = 5\times10^6$ to 10^{10}, where Re and Gr are based on the plate length L.

Wang also presented his data in the form of Nusselt versus Reynolds number for fixed Grashof numbers. When the data are replotted in the form of Nu/Nu_n versus the Reynolds number, it is found that, at $Gr = 5\times10^6$, Nu departs from Nu_n only beyond $Re = 2\times10^3$, but at $Gr = 10^{10}$ the departure occurs at $Re = 10^5$, so once again higher the Grashof number larger the Reynolds number range over which the natural convection heat transfer law is valid.

Wang's experimental data are replotted in the form Nu/Nu_n versus the internal Froude number Fr in Fig. 2. It is again seen that the two sets of data plot on virtually the same curve, demonstrating that Fr is indeed the appropriate parameter. From Fig. 2 we see that

$$Nu < 1.05 Nu_n \quad \text{if } Fr < (Fr_0 = 1.93).$$

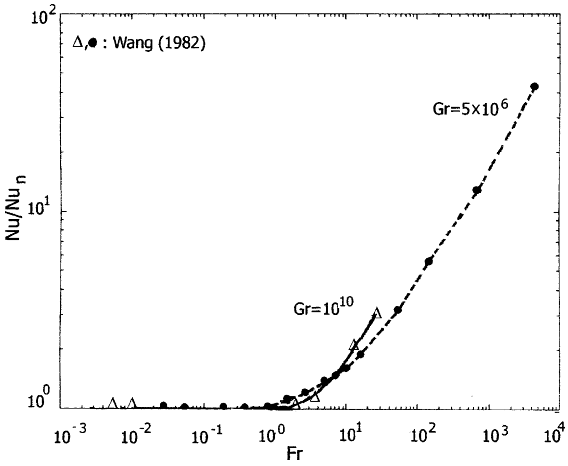

Fig. 2. Nusselt number ratio as a function of Froude number for upward facing heated flat plate.

2.3. Cylinder

Oosthuizen and Madan [15] carried out experiments on heat transfer from heated horizontal cylinders to an airstream in a vertical low-speed wind tunnel having a 0.4064 m × 0.4064 m working section. The experiments covered the Reynolds number range $Re = 10^2$ to 3×10^3 and Grashof number range $Gr = 2.5\times10^4$ to 3×10^5 (Re and Gr both being based on the diameter of the cylinder). Oosthuizen and Madan presented their data in the form Nu versus Re. These data again show that at $Gr = 3.75\times10^4$, Nu departs from Nu_n only beyond $Re = 100$, but at $Gr = 3\times10^5$ the departure occurs at $Re = 500$, confirming once again that higher the Grashof number larger the Reynolds number range over which the free convection heat transfer law is valid.

The data are replotted in the form Nu/Nu_n versus the Froude number in Fig. 3. Again the two sets of data plot

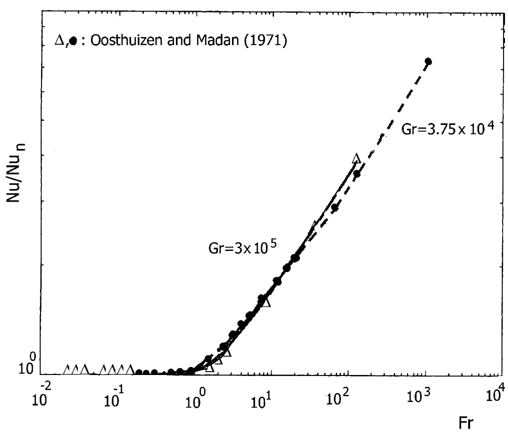

Fig. 3. Nusselt number ratio as a function of Froude number for cylinder in cross-flow.

on virtually the same curve, demonstrating that Fr is indeed the appropriate parameter. Furthermore

$$Nu < 1.05 Nu_n \quad \text{if } Fr < (Fr_0 = 1.65).$$

3. Discussion and conclusion

From the above results it is seen that for the heat flux to be given adequately by the law for natural convection, the Froude number should remain below a limiting value Fr_0, which depends on the body in the flow. (Interestingly, the determining parameter is a Froude number in all three cases considered here, although in the absence of buoyancy effects flow past the bodies considered includes both attached (flat plate) and separated (cylinder and sphere) regimes.)

We can equivalently say that the cross-flow velocity should be given by

$$U < U_0 = Fr_0 (g'L)^{1/2}.$$

We now collect our results on the velocity condition for the natural convection law to be valid (to within 5%):

sphere: $U < 0.063(gD|\Delta T|/T_r)^{1/2}$,
flat plate: $U < 1.93(gL|\Delta T|/T_r)^{1/2}$, and
cylinder: $U < 1.65(gD|\Delta T|/T_r)^{1/2}$.

A significant conclusion of the present analysis is that, for a given characteristic length scale, the limiting cross-flow velocity upto which heat transfer follows natural convection laws is far lower for three-dimensional than for two-dimensional flows.

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